Unified Cosmic Mechanics Evolution Theory (XVI) : Derivation and Verification of the Electron Dynamic Radius Formula

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Abstract

[**Series Information**] This paper is one of 23 installments in the Unified Cosmic Mechanics Evolution Theory. This framework is built upon the monumental achievements of the great scientists who preceded us. Its mission is to provide a foundational explanation of physical reality through the integration of Logic, Mathematics, and Empirical Observation. By introducing the Generalized Dynamical State Evolution Logic, this framework provides a compatibility reconciliation for classical mechanics, relativity, and quantum mechanics. Driven by natural and necessary evolutionary constraints,this framework resolves long-standing systemic conflicts, addressing core issues such as ultraviolet divergence, quantum uncertainty, the dark matter problem, wave-particle duality, the nature of mass-energy conversion, and conservation anomalies. Its scope extends from microscopic particles to macroscopic matter, and into the emergence of life and intelligence. We wish to state our position clearly: this framework does not negate the brilliant work of our predecessors. On the contrary, we believe the foundational observations and laws established by them are fundamentally correct. Our work is an effort to find a unified path of interpretation that honors their exceptional contributions while advancing our collective understanding. We express our deepest gratitude for the centuries of effort and wisdom that have paved the way for this synthesis.

[**This article**] This paper is the sixteenth in the 22-paper series of the “Unified Cosmic Mechanics Evolution Theory” framework. Grounded in fundamental dynamical evolutionary principles, the framework develops a unified physical description that is consistent across mathematical formalism, logical structure, and empirical phenomena, and provides a coherent reconstruction of classical mechanics, relativity, and quantum mechanics within a single relational evolution system.

Since Newton's classical mechanics established the concept of ”point mass”, simplifying objects into geometric points without volume or internal structure has become a powerful mathematical tool for physics to handle gravitational and motion problems. However, this idealized assumption has triggered a serious theoretical crisis in quantum electrodynamics (QED) — the infinite divergence of electromagnetic self-energy, the dilemma of artificial cutoff in renormalization, and the ontological ambiguity of the wave function collapse mechanism. Based on the Unified Cosmic Mechanics Evolution Theory, this paper proposes a new ontological perspective: everything in the universe is not composed of abstract ”points”, but of the only evolutionary carrier — the momentum unit (m0m0). Within this framework, all particles are composite entities formed by the vector superposition of multiple momentum units. For fermions (such as electrons), to maintain the stability and symmetry of the evolutionary system, their internal momentum flow must adopt a spherical symmetric encapsulation form, naturally endowing electrons with a clear intrinsic radius and completely abandoning the ”geometric point” assumption. The electron is neither a rigid point particle with a fixed state nor a dimensionless singularity, but an adaptive momentum encapsulation body with a dynamic radius R(p)R(p). This paper will strictly derive the electron dynamic radius formula, conduct self-consistency verification through the hydrogen atom system, and reveal the geometric and physical mechanism behind the dual characteristics of ”cloud state” and ”particle state” of micro-particles, providing a natural ultraviolet cutoff scheme for solving the QED divergence problem.

Keywords: Electron structure; Point particle; Dynamic particle radius; Wave-particle duality; Nature of quantum entanglement; Evolution theory; Momentum unit

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1 Introduction

Since Newton's classical mechanics established the concept of ”point mass”, simplifying objects into geometric points without volume or internal structure has become a powerful mathematical tool for physics to handle gravitational and motion problems. The historical evolution of this idealized assumption can be traced back to the early development of classical mechanics: to simplify the calculation of celestial motion and macroscopic object motion, Newton proposed the ”point mass” model, concentrating the mass of an object at a geometric point [1][2], which successfully solved a series of classical problems such as planetary orbits and falling body motion. With the rise of electromagnetism, this assumption was directly transplanted to simplify the calculation of charge interactions; in the early development of quantum mechanics, although the Dirac equation and quantum electrodynamics (QED) achieved remarkable success in predictions, the ontological assumption that ”the electron is a geometric point” triggered a serious theoretical crisis — the infinite divergence of electromagnetic self-energy, the dilemma of artificial cutoff in renormalization, and the ontological ambiguity of the wave function collapse mechanism [3][4][5][6]. For a long time, constrained by the historical inertia of the ”point particle” model, the physics community has often regarded these contradictions as flaws that can be handled by mathematical techniques, rather than defects of the model itself.

However, with the improvement of detection accuracy and the in-depth development of basic theories, returning to the real structure of particles has become an inevitable trend [7]. Based on the Unified Cosmic Mechanics Evolution Theory, this paper proposes a new ontological perspective: everything in the universe is not composed of abstract "points", but of the only evolutionary carrier — the momentum unit (m0m0). Within this framework, all particles are composite entities formed by the vector superposition of multiple momentum units [8], and their total evolutionary resource is strictly constrained between 00 and cc [9].

Especially for fermions (such as electrons), to maintain the stability and symmetry of the evolutionary system, their internal momentum flow must adopt a spherical symmetric encapsulation form. This encapsulation structure naturally endows electrons with a clear intrinsic radius, completely abandoning the "geometric point" assumption. More importantly, since the total evolutionary resource of an electron must be allocated in a zero-sum game between "internal spin maintenance" and "external spatial displacement", the oscillation amplitude of its internal momentum flow (i.e., the spatial projection radius) must dynamically evolve with the external motion state (velocity, momentum), among which Bohmian pilot wave theory and photoelectric causal interaction theory are typical logical representatives of this theory [10][11][12].

Therefore, the electron is neither a rigid point particle with a fixed state as considered in traditional theories [13][14], nor a dimensionless singularity, but an adaptive momentum encapsulation body with a dynamic radius R(p)R(p) [15]. This paper will clarify the core conditions of derivation, strictly derive the electron dynamic radius formula combined with the core axioms of the evolution theory and the relativistic interaction covariance constraints [16], conduct self-consistency verification through the hydrogen atom system, and based on the verification results analyze its theoretical predictive value, reveal the geometric and physical mechanism behind the dual characteristics of "cloud state" and "particle state" of microparticles, and provide a natural ultraviolet cutoff scheme for solving the QED divergence problem.

Note: This paper is derived based on multiple preceding chapters within the framework of this evolution theory, mainly verifying its logic, i.e., the radius of a particle dynamically changes with the environment and momentum deviation, and all particles are encapsulated by basic momentum units through topological coding and quantum entanglement (inline protocol), thereby forming overall cooperative evolution capability. For details, see multiple chapters such as quantum entanglement, fields and particles, and reconstruction of physical dimensions.

2 Derivation of the Electron Dynamic Radius Formula

2.1 Core Premises of Derivation (Based on the Axioms of the Unified Cosmic Mechanics Evolution Theory, Clarifying the Derivation Conditions)

The core conditions of the derivation in this paper are: everything in the universe takes the momentum unit as the only primitive; the electron, as a vector superposition of multiple momentum units, has conserved total evolutionary capacity and is constrained by the speed of light c; the internal momentum flow of fermions must satisfy spherical symmetry encapsulation to maintain stability; the derivation process strictly follows the requirements of special relativity covariance, does not introduce any artificial assumptions, and is only based on the first principles of the evolution theory and known physical laws. The specific core premises are as follows:

1. **Unique Primitive and Conservation of Evolutionary Capacity:** All material entities in the universe are composed of the only evolutionary carrier — the momentum unit (m_0). Each

momentum unit has a maximum evolutionary capacity of the speed of light c; a single electron, as a vector superposition of multiple momentum units, has a constant total evolutionary capacity of c within the unit cosmic reference time. This conservation law is the fundamental constraint for deriving the dynamic radius, determines the zero-sum game relationship between the internal structure of the particle and the external motion state, and is also one of the core conditions for the derivation in this paper.

2. **Necessity of Spherically Symmetric Encapsulation and Intrinsic Radius:** Based on the stability principle of the evolutionary system, the distribution of internal momentum flow of fermions must satisfy spherical symmetry to offset anisotropic stress and maintain the minimum energy state. This spherically symmetric encapsulation mechanism naturally endows electrons with a clear intrinsic geometric boundary (radius R), fundamentally negating the "point particle" assumption, and is also the core premise for deriving the intrinsic radius of electrons. The radius of an electron is essentially the maximum projection range of its internal momentum flow oscillation in three-dimensional space. This is consistent with the conclusion of modern experiments verifying the spherical symmetry of electrons [17][18][19].

3. **Covariance Constraints and Resource Allocation:** Since the upper limit of the evolution amplitude is c, the internal evolution speed (maintaining its own existence/mass, velocity component v_{in}) and the external motion speed (spatial displacement/momentum, velocity component v_{out}) of the electron satisfy the Pythagorean constraint, which is the geometric embodiment of the conservation of evolutionary capacity; combined with the special relativity covariance factor γ (external interaction window compression factor), $\gamma = 1/\sqrt{1-(v_{\text{out}}/c)^2}$, ensuring that the derivation results conform to the relativistic space-time characteristics, which is an important constraint condition for the derivation [20][21].

4. **Dynamic Correlation Between Radius and Internal Evolution:** The electron radius R is a direct spatial mapping of its internal evolution rate v_{in}. In the static state ($v_{\text{out}} = 0$), all evolutionary capacity is used for internal maintenance ($v_{\text{in}} = c$), and the electron is in the intrinsic relaxed state, corresponding to the maximum radius R_0. This scale is determined by the electron Compton wavelength λ_c ($R_0 = \lambda_c/2\pi$). The energy balance in this state exactly meets the threshold condition of "potential space particle replication" in this framework, explaining the consistency of the properties of similar electrons in the entire universe and their physical significance as the stable limit radius. In the moving state ($v_{\text{out}} > 0$), part of the evolutionary capacity is occupied by external displacement, leading to a decrease in the internal evolution rate v_{in}, which in turn causes a contraction of the oscillation amplitude of the internal momentum flow, manifested as a decrease in the dynamic radius $R(p)$. This logic is inherently consistent with the free electron zitterbewegung theory proposed by Schrödinger, whose zitterbewegung speed is exactly the speed of light c [22].

5. **Supplementary Correlation:** Photons are formed by the spiral winding and weaving of electrons; 360 degrees of the electron sphere corresponds to 180 degrees of the spin axis, and the spin is restored after 720 degrees; the photon wavelength is related to the electron radius; the electron radius is inversely proportional to the momentum deviation; the momentum units winding and weaving within the electron circumference form photons, and the momentum unit density is inversely proportional to the electron circumference (or radius) and proportional to the fine structure constant α. This correlation can be further explained in combination with the particle encapsulation characteristics of Yang-Mills gauge field theory [23].

2.2 Step-by-Step Derivation Process

1. **Step 1: Establish the relationship between internal evolution speed and external speed**

   According to the conservation of total evolutionary capacity and the Pythagorean constraint, the internal evolution speed v_{in} and the external motion speed v_{out} satisfy:

   $$v_{\text{in}}^2 + v_{\text{out}}^2 = c^2 \Longrightarrow v_{\text{in}} = \sqrt{c^2 - v_{\text{out}}^2} = c\sqrt{1 - (v_{\text{out}}/c)^2} = c/\gamma$$

   where $\gamma = 1/\sqrt{1 - (v_{\text{out}}/c)^2}$ is the special relativity covariance factor (external interaction window compression factor).

2. **Step 2: Establish the correlation between electron radius and internal evolution speed**

   From Core Premise 4, the electron radius R is proportional to the internal evolution speed v_{in}, i.e., $R(v_{\text{out}}) \propto v_{\text{in}}$.

   Combined with the maximum static radius $R_0 = \lambda_c/2\pi$ (when $v_{\text{in}} = c$), the proportional relationship is obtained:

   $$R(v_{\text{out}})/R_0 = v_{\text{in}}/c$$

3. **Step 3: Substitute the covariance factor to obtain the velocity-dependent radius formula**

   Substitute $v_{\text{in}} = c/\gamma$ and $R_0 = \lambda_c/2\pi$ into the above proportional formula, and the derivation gives:

   $$R(v_{\text{out}}) = R_0 \cdot (1/\gamma) = (\lambda_c/2\pi)/\gamma$$

4. **Step 4: Convert the velocity v_{out} to momentum p to obtain the final momentum-dependent formula**

   According to the relationship between relativistic momentum and γ, $P = m_0\gamma v_{\text{out}}$ (m_0 is the electron rest mass), combined with the relativistic energy-momentum relationship $E^2 = (pc)^2 + (m_0c^2)^2$ and $E = \gamma m_0 c^2$, the relationship between γ and momentum p is derived by combining:

   $$(\gamma m_0 c^2)^2 = (pc)^2 + (m_0c^2)^2 \Longrightarrow \gamma^2 = (p/(m_0c))^2 + 1$$

   Let $p_0 = m_0c$ (the momentum threshold corresponding to the electron rest mass), then $\gamma = \sqrt{1 + (p/p_0)^2}$. Substitute it into the radius formula in Step 3 to obtain the final electron dynamic radius formula:

   $$R(p) = (\lambda_c/2\pi)/\sqrt{1 + (p/p_0)^2}$$

5. **Step 5: Supplementary correlation formula (electron radius and photon wavelength)**

   Based on the weaving relationship between electrons and photons, combined with the fine structure constant α (1/137), two supplementary formulas are derived:

   Electron radius: $R = \lambda_a/2\pi$ (λ_a is the electron intrinsic characteristic wavelength, i.e., the Compton wavelength λ_c)

   Photon wavelength: $\lambda =$ electron spherical circumference$/\alpha = 2\pi R/\alpha$

Supplementary logic: The fine structure constant α (1/137) represents the compression ratio of the number of momentum units within the wavelength of orbital quantization; let n be the number of compressed m_0 within the radius in a single direction of the electron sphere, then the wavelength $\lambda = (1/137n) \cdot m_0$. The larger n is, the shorter the wavelength and the smaller the electron radius. This logic can be further verified in combination with the correlation between photon wavelength and electron scale in Compton scattering experiments [24].

3 Verification of the Electron Dynamic Radius Formula (Self-Consistency Test in the Hydrogen Atom System)

3.1 Core Verification Hypothesis and Parameter Definition

Core verification hypothesis: The electron is not a mathematical point particle, but a physical entity with a momentum-dependent intrinsic radius, whose radius is defined by the formula $R(p) = (\lambda_c/2\pi)/\sqrt{1+(p/p_0)^2}$. The verification process strictly follows the core conditions derived in this paper to test the self-consistency and theoretical predictive ability of the formula.

Definition of key parameters:

- p: Instantaneous relativistic momentum of the electron
- $p_0 = mc$: Momentum threshold corresponding to the electron rest mass (natural unit)
- $\lambda_c = h/(mc) \approx 2.426 \times 10^{-12}$m: Electron Compton wavelength (electron intrinsic geometric scale)
- $\alpha \approx 1/137.036$: Fine structure constant (coupling strength)

Reference data (hydrogen atom ground state $n = 1$):

- Electron orbital momentum $p_1 = \alpha mc = \alpha p_0$ (much smaller than p_0)
- Bohr radius $a_0 = \hbar/(\alpha mc) \approx 5.29 \times 10^{-11}$m (orbital scale)
- Classical electron radius $r = \alpha \cdot \lambda_c/(2\pi) \approx 2.82 \times 10^{-15}$m (electromagnetic coupling scale)

Key distinction: Strictly distinguish between the electron intrinsic radius (R, the scale of the particle itself) and the orbital radius (a_0, the scale of the electron motion orbit). The core of the verification is to confirm the reasonable description of $R(p)$ on the size of the electron itself, and to test whether the theoretical prediction of the formula is consistent with the known experimental phenomena.

3.2 Scenario-Based Verification Process (Combined with Theoretical Prediction Test)

1. **Scenario 1: Low-Energy Bound State (Hydrogen Atom Ground State $n = 1$)**

   Input conditions: In the ground state of the hydrogen atom, the electron velocity $v_1 = \alpha c$, so the momentum ratio is $p_1/p_0 = \alpha \approx 1/137$.

   Formula calculation: Substitute p_1 into the electron dynamic radius formula. Since $\alpha^2 \approx 5.3 \times 10^{-5} \ll 1$, the denominator $\sqrt{1+\alpha^2} \approx 1$, so:

$$R(p_1) = (\lambda_c/2\pi)/\sqrt{1+\alpha^2} \approx \lambda_c/2\pi \approx 3.86 \times 10^{-13}\text{m} = 386\text{fm}$$

Result analysis and theoretical prediction verification:

(a) **Verification of correlation with Compton wavelength:** The calculation result converges to $\lambda_c/2\pi$ (reduced Compton wavelength), verifying the theoretical prediction that "electrons maintain the intrinsic relaxed radius in the low-energy state", and supporting the hypothesis that "electrons are non-point particles but finite-size wave packets". At the same time, the energy balance state corresponding to this intrinsic radius meets the threshold condition of "potential space particle replication", explaining the phenomenon of consistent properties of similar electrons in the entire universe, forming self-consistency with the core premises derived in this paper, and verifying the rationality of the premise assumptions. This conclusion is consistent with the view proposed by Hestenes that electron spin originates from internal light-speed circular motion [25].

(b) **Verification of comparison with classical electron radius:** $R \approx 386$fm, which is about 137 times the classical electron radius r (2.82fm), verifying the theoretical prediction of this paper that "the classical electron radius is not the real physical size of the electron, but the effective coupling length of electromagnetic interaction", indicating that $\lambda_c/2\pi$ is the real geometric boundary of electron momentum unit encapsulation, providing a natural explanation for the self-energy divergence problem caused by the point particle model. This is consistent with the view proposed by Barut et al. that "electrons are structured extended objects" [26].

(c) **Verification of comparison with Bohr radius:** $a_0 \approx 52900$fm, $R \approx 386$fm, the ratio $a_0/R \approx 137 \approx 1/\alpha$, verifying the prediction that "the intrinsic size of the electron is much smaller than its motion orbit, but cannot be ignored at the atomic scale", which can provide a natural cutoff mechanism for the Lamb shift, forming a response to the QED experimental observation results [6].

2. **Scenario 2: High-Energy Limit ($p \gg p_0$)**

Input conditions: The electron is accelerated to extremely high energy, satisfying $p \gg mc$ (such as the TeV collider energy region).

Formula calculation: At this time, the denominator $\sqrt{1+(p/p_0)^2} \approx p/p_0$, substituting into the formula gives:

$$R(p) = (\lambda_c/2\pi)/\sqrt{1+(p/p_0)^2} \approx (\lambda_c/2\pi) \cdot (m_0 c/p)$$

Combined with $\lambda_c = h/(m_0 c)$, further derivation gives:

$$R(p) \approx h/(2\pi p) = \hbar/p$$

which is consistent with the reduced form of the de Broglie wavelength.

Result analysis and theoretical prediction verification:

(a) Verification of particle-wave unification prediction: In the high-energy limit, the intrinsic geometric radius of the electron can shrink to be close to the reduced de Broglie wavelength, verifying the theoretical prediction of this paper that "electrons can realize the unification of particle nature (radius) and wave nature (wavelength)", providing a geometric dynamic explanation for wave-particle duality. This is consistent with the relativistic wave characteristics of electrons revealed by the Dirac equation [4].

(b) Corresponding verification with high-energy scattering experiments: It is highly consistent with the phenomena of high-energy scattering experiments — as the energy increases, the effective scattering cross-section of electrons decreases ($\sigma \propto 1/p^2$), showing a behavior similar to "asymptotic freedom". This verifies the prediction of this paper that "electrons are compressed to an extremely small scale by momentum at high energy", indicating that the observed characteristics of electrons at high energy are similar to point particles, which are actually instantaneous compressed states, forming a complement to the intrinsic relaxed radius at low energy without conflict, further verifying the rationality of the dynamic radius formula. This is consistent with the law of relativistic Compton scattering cross-section described by Klein and Nishina [27].

3.3 Theoretical Self-Consistency and Unique Predictions (Prediction Extension Based on Verification Results)

1. **Solving scale hierarchy confusion:** Clearly define the two core scales of electrons, which are naturally correlated through the fine structure constant α, forming a logical closed loop: the micro core $R \sim \lambda_c/2\pi$ (about 386fm), determined by momentum unit encapsulation; the macro orbit $a_0 \sim \lambda_c/\alpha$ (about 53000fm), determined by Coulomb potential well and angular momentum quantization, verifying the rationality of scale division and avoiding the scale confusion problem of traditional theories. This is consistent with the view pointed out by Cao Tianyu in his study on the development of field theory concepts in the 20th century that "there is a scale confusion problem in the transition from the finite radius model to the point particle model" [28].

2. **Providing a natural ultraviolet cutoff:** The "self-energy divergence" problem in quantum electrodynamics (QED) originates from the assumption that electrons are point particles ($R \to 0$). In this theory, the minimum radius of electrons is $\lambda_c/2\pi$ (low-energy state) or $\hbar/p$ (high-energy state), which is never zero. When calculating the electron self-energy or Lamb shift, the lower limit of the integral can be naturally cut off at R, without the need to artificially introduce renormalization cutoff parameters, eliminating infinities geometrically. This prediction can be further verified through high-precision self-energy calculation experiments. This idea is complementary to the renormalization scheme proposed by Schwinger, avoiding the theoretical controversy of artificial cutoff [6].

3. **Predicting the nature of the classical electron radius:** The theory predicts that the classical electron radius r is not the real physical size of the electron, but the interaction characteristic length modified by the coupling constant α, and the real size $R \approx 137 \times r$. This prediction can be further tested through high-precision atomic spectroscopy experiments or extremely high-precision electron scattering experiments, providing a clear direction for subsequent experimental research. This is consistent with the view proposed by Wilczek that "mass is dynamic encapsulation" [29].

4 Supplementary Logic and Formula Explanation

4.1 Overall Logical Chain

The greater the momentum deviation → the smaller the electron radius → the shorter the photon wavelength → the greater the momentum unit density → the greater the total number of momentum units (mass number) → the greater the energy mc^2, forming a self-consistent physical logical closed loop,

which is completely consistent with the core premises, dynamic radius formula and verification results derived in this paper. This logical chain can be further corroborated in combination with the energy relationship $E = mc^2$ established by Einstein [21].

4.2 Explanation of Two Alternative Formulas

Based on the logic of the original text, two electron radius formulas are provided, which are suitable for different theoretical assumptions and both meet the core conditions derived in this paper:

1. **Smooth transition model (core formula):** $R(p) = (\lambda_c/2\pi)/\sqrt{1 + (p/p_0)^2}$. Physical meaning: In the low-energy limit ($p \ll p_0$), $R \approx \lambda_c/2\pi$ (electrons maintain the intrinsic radius); in the high-energy limit ($p \gg p_0$), $R \propto 1/p$ (radius is inversely proportional to momentum); p_0 is the characteristic momentum threshold, marking the turning point where electrons enter the significantly compressed state from the relaxed state, which is consistent with the behavior of most physical systems. The radius in the low-energy limit meets the potential space particle replication condition, ensuring the consistency of the properties of similar electrons.

2. **Extreme simplification model:** $R(p) = k/p$ (k is a constant). Physical meaning: The radius is strictly inversely proportional to the momentum; if $k = \hbar$, then $R = \hbar/p$, implying that electrons have no fixed intrinsic radius, and their size is completely determined by the momentum state; when the momentum is infinite, the radius approaches 0, which is a more radical theoretical prediction and can be used as a reference for subsequent theoretical expansion.

4.3 Supplementary Notes on Photons

Photons follow $f\lambda = c$ (f is the macro transition cycle frequency, λ is the macro occupation scale of photons or the transition scale of a single macro time window). Regardless of the distribution of wavelength and frequency, their product is always c; photons adopt goose formation coding, and their wavelength is related to the electron radius. The smaller the electron radius, the shorter the wavelength of the excited photon and the greater the momentum deviation; when the electron stable radius is the smallest (reduced Compton wavelength), the excited photon is extreme ultraviolet light, which carries the maximum energy that can shape the space-time state, and the energy condition corresponding to this stable radius exactly meets the requirement of replicating new electrons in the potential space. This supplement can be further understood in combination with the interaction law between photons and electrons in Compton scattering experiments [24].

5 Relative Compatibility Test of the Two Wavelength Formulas for Electron Radius

In the previous derivation, the electron radius formula $R = \lambda_a/2\pi$ (λ_a is the electron intrinsic characteristic wavelength) and the covariance radius formula $R(p) = (\lambda_c/2\pi)/\sqrt{1 + (p/p_0)^2}$ are mentioned. The relative compatibility of the two is a key test point for theoretical self-consistency: if they are not relatively compatible, the theoretical system will have logical cracks; if they are relatively compatible, they can complement each other, describing the "static intrinsic structure" and "dynamic evolution behavior" of electrons respectively.

Conclusion: The two formulas are relatively compatible, showing a dialectical relationship of "substance" and "function" — $R = \lambda_a/2\pi$ defines the "intrinsic geometric scale" (reference radius R_0) of electrons in the static state or specific energy level; the covariance formula describes the "dynamic evolution law" of this intrinsic scale with velocity/momentum. The combination of the two can obtain a relatively complete electron dynamic radius theory, forming a unified whole with the derivation conditions and verification results of this paper.

5.1 In-depth Analysis: The Physical Roles of the Two Formulas

1. **Formula A:** $R_0 = \lambda_a/2\pi$ — The essence of geometric definition. Source: This is the geometric embodiment of wave-particle duality in quantum mechanics. For a standing wave in circular motion, its circumference $C = 2\pi R$ must be an integer multiple of the wavelength λ (1 time for the ground state), i.e., $2\pi R = \lambda \Longrightarrow R = \lambda/2\pi$. Physical meaning: This formula clearly shows that electrons are not abstract mathematical point particles, but finite geometric entities supported by wave nature. Among them, λ_a corresponds to the electron Compton wavelength λ_c in the electron static state, so the static reference radius (intrinsic radius) of the electron is actually the reduced Compton wavelength: $R_0 = \lambda_c/2\pi = h/(2\pi m_e c) = \hbar/(m_e c) \approx 3.86 \times 10^{-13}$m. Confirmation of relative compatibility: This formula establishes that the electron intrinsic radius $R_0 \neq 0$, providing the core geometric parameter for the "electron non-point particle" and "natural cutoff of self-energy divergence" proposed earlier, which is consistent with the theoretical premises and derivation conditions of the covariance formula, and also echoes the energy condition of potential space particle replication. This geometric definition is consistent with the geometric evolution characteristics of quantum states revealed by Berry phase [30].

2. **Formula B:** $R(v) = R_0/\gamma = (\lambda_c/2\pi)/\sqrt{1 + (p/p_0)^2}$ — The law of dynamic evolution. Source: Derived based on the "conservation of evolutionary capacity" and special relativity covariance constraints, $\gamma = 1/\sqrt{1 - (v/c)^2}$ is the covariance contraction factor. Physical meaning: When an electron obtains the external motion velocity v (or momentum p), its internal evolution resources are occupied by external interactions, leading to a decrease in the internal evolution rate, which in turn causes the spatial projection (intrinsic radius) of the electron to undergo covariant contraction. This formula describes the dynamic change law of the electron intrinsic radius R_0 with the external state (velocity/momentum), and is the expression of the "instantaneous effective radius" of electrons in different energy states. When the electron momentum approaches 0, the instantaneous effective radius approaches the intrinsic radius corresponding to the reduced Compton wavelength, satisfying the energy balance and particle replication conditions. This dynamic evolution law is inherently related to the length contraction effect proposed by Lorentz [20].

5.2 Relative Compatibility: Derivation of the Complete Dynamic Radius Formula

The core of the fusion of the two formulas is to clarify the physical connotation of λ_a — λ_a is the intrinsic characteristic wavelength (Compton wavelength) of electrons, not the de Broglie wavelength that changes with the motion state. The specific fusion process is as follows: Taking the Compton wavelength λ_c as the electron intrinsic characteristic wavelength, the static reference radius $R_0 = \lambda_c/2\pi$ (fixed constant, the "intrinsic size" of electrons) is obtained from Formula A; when the electron moves, constrained by covariance, its effective radius contracts with the γ factor, and substituting into the covariance relationship of Formula B, the complete dynamic radius formula is obtained:

$$R(p) = R_0/\sqrt{1+(p/p_0)^2} = (\lambda_c/2\pi)/\sqrt{1+(p/p_0)^2}$$

Supplementary note: If λ_a is mistakenly understood as the de Broglie wavelength $\lambda_{\mathrm{dB}} = h/p = h/(\gamma m_e v)$ that changes with velocity, forcing the application of $R = \lambda_a/2\pi$ will lead to $R = h/(2\pi\gamma m_e v)$, which is inconsistent with the form of the covariance formula. The essential reason is that $R = \lambda_a/2\pi$ is only applicable to the reference frame of the internal spin standing wave of electrons, describing the intrinsic geometric characteristics, while the de Broglie wavelength is the wave characteristic observed externally of electrons. The two are applicable to different scenarios and cannot be confused. Appropriate expression form: Correlate the dynamic radius with the effective intrinsic wavelength, i.e., $R(p) = \lambda_{\mathrm{effective}}/2\pi$, where $\lambda_{\mathrm{effective}} = \lambda_c/\gamma$, which means that the faster the electron moves, the lower the internal evolution rate, the shorter the effective intrinsic wavelength, and the smaller the corresponding effective radius, matching the core premise of "conservation of evolutionary capacity". This distinction can be further deepened in combination with Dirac's quantum electron theory [4].

5.3 Unification of Physical Images: The Core Significance of the 2π Factor

The 2π factor in Formula A is not accidental, but the key to connecting quantum mechanics and relativity, which can keep the theory consistent with experiments: Correlating the spin quantum number: $\hbar = h/2\pi$ is the minimum unit of angular momentum. $R_0 = \hbar/(m_e c)$ means that when the tangential velocity at the edge of the electron is c in the static state, the angular momentum is exactly $\hbar/2$ (electron spin quantum number), indicating that this geometric formula can provide an explanation for the geometric origin of electron spin, rather than a simple geometric guess. The key to eliminating self-energy divergence: When calculating the electron electromagnetic self-energy, if the lower limit of the integral is taken as $R = \lambda_c$, there will be a coefficient deviation in the result; if $R = \lambda_c/2\pi$ (reduced Compton wavelength) is taken, the calculated self-energy value $E \approx e^2/(4\pi\varepsilon_0 R)$ can be close to the electron rest mass energy $m_e c^2$ (depending on the charge distribution model), making the theoretical prediction consistent with the experimental value, further indicating the rationality of the fusion of the two formulas, and also consistent with the energy condition of potential space particle replication. This conclusion is consistent with Dirac's thinking on electron structure in his proposed classical electron radiation theory [5].

5.4 Summary of Relative Compatibility

$R = \lambda_a/2\pi$ and the covariance radius formula are complementary: the former defines the "intrinsic identity" (reference geometric scale) of electrons, and λ_a corresponds to the Compton wavelength; the latter defines the "survival state" (dynamic evolution scale) of electrons, describing the contraction law of the radius with momentum by the covariance factor. The combination of the two can realize the unification of quantum mechanical wave nature (λ) and relativistic space-time nature (γ) on the electron geometric entity, improving the theoretical system of electron dynamic radius. Among them, the intrinsic radius corresponding to the reduced Compton wavelength is the key link connecting the two, which not only establishes the reference of the electron intrinsic scale, but also meets the core conditions of potential space particle replication, explaining the phenomenon of consistent properties of similar electrons. This unified perspective is consistent with Hestenes' idea of reconstructing quantum mechanics with geometric algebra [25].

6 Significance of Paradigm Shift: From "Mathematical Point" to "Dynamic Encapsulation Body"

The derivation process of this paper clearly shows the leap from the traditional "point particle" paradigm to the "momentum unit dynamic encapsulation" paradigm. Due to the adoption of the point mass idealization since Newton, traditional theories have been forced to introduce complex renormalization techniques to repair the infinite loophole [1][6]; based on the Unified Cosmic Mechanics Evolution Theory, this paper starts from the first principles, takes "momentum unit as the only primitive, conservation of evolutionary capacity, and spherical symmetry encapsulation" as the core derivation conditions, and proves that the radius is the inevitable geometric result of momentum allocation. The electron is no longer an abstract point passively accepting observation, but an intelligent evolutionary system that actively adjusts its own geometric scale ($R(p)$) according to the environment (momentum input). This shift in perspective not only naturally eliminates self-energy divergence, but also provides a unified geometric dynamic explanation for understanding the wave-particle duality of micro-particles, Lamb shift, and even vacuum fluctuations, providing a new idea for the research of micro-particle physics. This paradigm shift is consistent with the view emphasized by Jammer in his study on the evolution of the concept of mass that "physical models should be consistent with the real structure" [31].

7 Conclusion

The electron dynamic radius formula $R(p) = (\lambda_c/2\pi)/\sqrt{1 + (p/p_0)^2}$ has passed preliminary numerical and logical verification in the hydrogen atom system, which is basically consistent with the experimental logic, and strictly follows the derivation conditions established in this paper, with the following core characteristics:

1. **Low-energy consistency:** In the atomic bound state, it can restore the intrinsic radius of the electron's reduced Compton wavelength, supporting the non-point characteristics of electrons; this intrinsic radius meets the potential space particle replication condition, ensuring the consistency of the properties of similar electrons in the entire universe, and may replicate new electrons, which is highly consistent with the verification results. This is consistent with the modern experimental verification results of electron spherical symmetry [17][18][19].

2. **High-energy asymptoticity:** In the relativistic limit, it can transition to the reduced de Broglie wavelength, realizing the unification of particle geometry and wave properties; at high energy, electrons can be detected to an extremely small scale of 10^{-18}m, which is an instantaneous state after momentum compression, forming a complement to the intrinsic radius at low energy without conflict, and consistent with the phenomena of high-energy scattering experiments [27].

3. **Logical completeness:** It clearly distinguishes between the electron intrinsic radius and the orbital radius. Through the relative compatible fusion of the two wavelength formulas, it provides a geometric basis for solving the QED self-energy divergence problem, and forms a self-consistent logic with the weaving relationship of photons, and the relevant predictions can be tested through experiments, with strong theoretical self-consistency and verifiability. This logical completeness can be further corroborated in combination with Yang-Mills gauge field theory, Berry phase and other theories [23][30].

Summary: According to this evolution theory, the complexity of observed particles originates from

multiple factors. Micro-particles are different from macro-objects, and their specific shapes cannot be directly observed through surface reflection; at the same time, micro-particles are derived within this framework to have a radius that contracts with the decrease of input momentum deviation, making it difficult to observe their position and size, and the larger the radius, the more diffuse they are. Combined with spin and orbital motion, they will show a chaotic cloud state. At the same time, particles have three phase transitions: spherical collapse state, spherical diffuse state, and fluid state, which brings certain difficulties to the research of micro-particles. Therefore, we can only infer the behavior and rules of micro-particles as a whole through the relevant logic of the macro framework. The electron radius formula within this framework is a pre-fitting based on experimental results and relevant theories within the evolution theory framework, and its accuracy still needs to be strictly verified by experiments. This summary is consistent with Cao Tianyu's philosophical reflection on the development of field theory concepts [28].

References

[1] Newton, I. (1687). Philosophiæ Naturalis Principia Mathematica. London: Royal Society (ed. by E. Halley). — The mathematical origin of the point mass concept, proving that the gravity of a spherically symmetric object is equivalent to the mass concentrated at the geometric center.

[2] Westfall, R. S. (1983). Never at Rest: A Biography of Isaac Newton. Cambridge: Cambridge University Press. — The biography discusses Newton's true views on the corpuscular theory, distinguishing between mathematical tools and physical reality.

[3] Kirk, G. S., Raven, J. E., & Schofield, M. (1983). The Presocratic Philosophers (2nd ed.). Cambridge: Cambridge University Press. — Detailed description of Democritus' atoms with shape and size, establishing the "discrete basic unit" ontology.

[4] Dirac, P. A. M. (1928). The quantum theory of the electron. Proceedings of the Royal Society of London Series A: Mathematical, Physical and Engineering Sciences, 117(778), 610–624. (DOI: 10.1098/rspa.1928.0023). — The authoritative source of point particles, but proves that spin 1/2 and magnetic moment are natural results of the relativistic wave equation, implying the underlying geometric mechanism.

[5] Dirac, P. A. M. (1938). Classical theory of radiating electrons. Proceedings of the Royal Society of London Series A, 167(930), 148–169. (DOI: 10.1098/rspa.1938.0124). — Reveals the pathological state of point particles in classical electrodynamics, such as pre-acceleration phenomenon.

[6] Schwinger, J. (1948). On quantum-electrodynamics and the magnetic moment of the electron. Physical Review, 73(4), 416–417. (DOI: 10.1103/PhysRev.73.416). — Renormalization establishes the point particle paradigm, achieving predictive success by artificially canceling infinities.

[7] Xiao, Bo. (forthcoming). Unified Cosmic Mechanics Evolution Theory (XI): Fields and Particles — Momentum Topological Coding Deterministic Quantum Theory [In Press].

[8] Xiao, Bo. (forthcoming). Unified Cosmic Mechanics Evolution Theory (XIX): Momentum Flow Allocation Integral Principle of Multi-Slit Experiment [In Press].

[9] Xiao, Bo. (forthcoming). Unified Cosmic Mechanics Evolution Theory (XII): Compatibility Verification of Momentum Encapsulation Speed Increase Effect [In Press].

[10] Bohm, D. (1952). A suggested interpretation of the quantum theory in terms of "hidden" variables. I & II. Physical Review, 85(2), 166–193. (DOI: 10.1103/PhysRev.85.166). — The derivation proves that particles have physically real trajectories, indirectly proving physical reality.

[11] Feynman, R. P. (1949). Space-time approach to quantum electrodynamics. Physical Review, 76(6), 769–789. (DOI: 10.1103/PhysRev.76.769). — Although it is a point particle framework, it implies the corresponding causal relationship between light and electricity.

[12] Xiao, Bo. (forthcoming). Unified Cosmic Mechanics Evolution Theory (XV): Corresponding Relationship between Photon-Electron Causal Interaction State Evolution [In Press].

[13] Feynman, R. P. (1961). The Theory of Fundamental Processes. New York: W.A. Benjamin. — Textbook establishment and promotion of the point particle paradigm.

[14] Abraham, M. (1902). Dynamik des Elektrons. Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, 20–41. — Early finite radius electron model, assuming electrons are rigid spheres.

[15] Gabrielse, G., et al. (2006). Precision measurement of the positron g-2. Physical Review Letters, 97(3), 030801. (DOI: 10.1103/PhysRevLett.97.030801). — The experiment only gives the upper limit of the radius, not confirming that the radius is strictly zero.

[16] Xiao, Bo. (forthcoming). Unified Cosmic Mechanics Evolution Theory (VI): The Relationship between Relativity, Classical Mechanics and Quantum Mechanics [In Press].

[17] Hudson, J. J., et al. (2011). Improved measurement of the shape of the electron. Nature, 473(7348), 493–496. (DOI: 10.1038/nature10104). — Assists in supporting the spherical symmetry of electrons.

[18] ACME Collaboration. (2018). Improved limit on the electric dipole moment of the electron. Nature, 562(7727), 355–360. (DOI: 10.1038/s41586-018-0599-9). — Extremely high-precision verification of electron spherical symmetry, excluding chaotic or asymmetric structures.

[19] Baron, J., Campbell, W. C., Hutzler, N. R., et al. (2014). Order of magnitude smaller limit on the electric dipole moment of the electron. Science, 343(6168), 269-272. DOI: 10.1126/science.1248213 — This experiment confirms that electrons are perfectly spherical under extremely high standards, providing key experimental evidence for the core hypothesis of this paper that "electrons are momentum unit spherically symmetric cancellation encapsulation bodies", and supporting the view that there is a huge momentum flow (mc) inside electrons.

[20] Lorentz, H. A. (1904). Electromagnetic phenomena in a system moving with any velocity smaller than that of light. Proceedings of the Royal Netherlands Academy of Arts and Sciences, 6, 809–831. — Lorentz-Abraham model, the predecessor of relativity, deriving length contraction based on finite radius.

[21] Einstein, A. (1905). Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig? Annalen der Physik, 323(18), 639–641. (DOI: 10.1002/andp.19053231814). — Establishes the equation $E = mc^2$, indicating mass conservation and each mass contributing c dynamic evolutionary capacity, which is revised to the space-time state shaping equation in this evolution theory.

[22] Schrödinger, E. (1930). Über die kräftefreie Bewegung in der relativistischen Quantenmechanik. Sitzungsberichte der Preußischen Akademie der Wissenschaften, Physikalisch-Mathematische Klasse, 24, 418–428. — Proposes that free electrons have zitterbewegung of $2mc^2/h$, with speed equal to the

speed of light c, which is consistent with the logic of "internal evolution speed" and "conservation of total evolutionary capacity".

[23] Yang, C. N., & Mills, R. L. (1954). Conservation of isotopic spin and isotopic gauge invariance. Physical Review, 96(1), 191–195. (DOI: 10.1103/PhysRev.96.191). — Original paper, although not explicitly stating the geometric meaning, it is the source of particle encapsulation characteristics and gauge field theory.

[24] Compton, A. H. (1923). A quantum theory of the scattering of X-rays by light elements. Physical Review, 21(5), 483–502. (DOI: 10.1103/PhysRev.21.483). — Proves that electrons have a characteristic scale, not point particles, and reveals the correlation between photon wavelength and electron scale.

[25] Hestenes, D. (1990). The zitterbewegung interpretation of quantum mechanics. Foundations of Physics, 20(10), 1213–1232. (DOI: 10.1007/BF01889458). — Reconstructs quantum mechanics with geometric algebra, clearly proposing that electron spin and mass originate from internal light-speed circular motion, directly supporting $R_0 = \lambda_C/2\pi$ and the topological coding view.

[26] Barut, A. O., & Van Huele, J. (1990). Self-field quantum electrodynamics: The two-level atom. Physical Review A, 41(5), 2284–2291. (DOI: 10.1103/PhysRevA.41.2284). — Holds that electrons are structured extended objects, and the photon field is part of the electron self-field, challenging the point particle assumption.

[27] Klein, O., & Nishina, Y. (1929). Über die Streuung von Strahlung durch freie Elektronen nach der neuen relativistischen Quantendynamik von Dirac. Zeitschrift für Physik, 52, 853–868. (DOI: 10.1007/BF01366547). — Describes the relativistic Compton scattering cross-section, consistent with the Coulomb force perceived cross-section attenuation and relativistic covariance.

[28] Cao, T. Y. (1997). Conceptual Developments of 20th Century Field Theories. Cambridge: Cambridge University Press. — Discusses the historical process of the transition from the finite radius model to the point particle model and the philosophical controversies of renormalization.

[29] Wilczek, F. (1999). Mass without mass. Physics Today, 52(11), 11–17. (DOI: 10.1063/1.882634). — Supports the view that mass is dynamic encapsulation.

[30] Berry, M. V. (1984). Quantal phase factors accompanying adiabatic changes. Proceedings of the Royal Society of London Series A, 392(1802), 45–57. (DOI: 10.1098/rspa.1984.0023). — Berry phase, the geometric phase acquired by quantum states during cyclic evolution in parameter space, supporting the geometric origin of spin.

[31] Jammer, M. (1961). Concepts of Mass in Contemporary Physics and Philosophy. Princeton: Princeton University Press. — Specializes in discussing the historical evolution of the concept of mass and the point particle model, an authoritative historical source.